

Dynamic macroscopic simulation of on-street parking search: a trip-based approach

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Abstract

This paper extends a trip-based aggregate dynamic traffic model to account for on-street parking search. The trip-based approach for a road network defined as a reservoir characterizes the internal traffic states by a macroscopic fundamental diagram (MFD) in speed while individualizing all vehicle travel distances. This paper first investigates distances to park for on-street parking based on real data in Lyon (France) and stochastic numerical experiments. An updated formulation compared to the existing literature is proposed for the relation between such distances and the parking occupancy. This new formulation is then incorporated into an event-based numerical scheme that solves the trip-based MFD model. The complete framework is able to account for different vehicle categories with respect to their parking strategies and to finely tune the related travel distances. Finally, the capabilities of the full framework are illustrated based on three different scenarios. The first two correspond to strategies with static and dynamic (reactive) switch of the demand from on- to off-street parking. While being very classical, they permit to demonstrate that the proposed model reacts as expected in such cases. The third scenario assesses the effect of a smart-parking technology that informs the users when a free parking spot is available on one of the downstream links at each intersection. In such a case, the model permits to estimate the benefit for the equipped users but also the impacts on all other vehicle categories. The three scenarios highlight that the proposed framework is versatile and can quickly provide a first assessment with a low calibration burden of different parking strategies or policies.

Introduction

Advances in driver information systems make parking search on off-street facilities easier. However, looking for an on-street parking spot when the number of available places is limited, mostly requires luck and patience. People have to drive randomly until a free place is reached. During peak hours, such trip-endings increase the total number of vehicle driving and thus decrease the level of service for other drivers. Several experimental studies show that this phenomenon is far from negligible. (Shoup, 2011) concludes from a survey in 16 American and European cities that 8% of the total traffic in a city corresponds to cars looking for free parking spaces. This can raise to 30% in business district. Another study from the same author (Shoup, 2006) shows that 8 to 74% of the total traffic is searching for parking and that the time required to reach a spot is usually between 3,5 and 14 min during the peak hours. These values are consistent with another study (Axhausen *et al.*, 1994) based on surveys in different European cities (Frankfurt, Birmingham, Kingston-upon-Times). The latter shows that between 4,9% and 40,3% of the total travel time in a city is dedicated to parking search. Some studies also demonstrate that not only parking searches increase the number of circulating vehicles but may also have a strong impact on speeds. For example, (Carrese *et al.*, 2004) mentions that drivers reduce their speed from 20 to 25 km/h when they start looking for a parking spot and further reduce their speed to 15 km/h when they are actively searching in dense environment. On the other hand, traffic conditions (and speeds) also influence the search process and it takes more time to screen the parking spots at lower speed.

If parking and traffic conditions have obvious and strong mutual interactions, few attention is paid to parking in simulation studies except when parking is the main purpose. This is even more true for on-street parking, which is quite complex to implement and tune in microscopic traffic simulators. In the recent years, several attempts have been made to provide an aggregated modelling of traffic flow including off- and on-street parking. (Arnott and Inci, 2006) and (Arnott and Inci, 2010) propose models inspired by economic studies to assess the performance of parking system on traffic congestion. The approach is interesting but rather limited for heavily congested road network because only stationary-state conditions are considered. (Zakharenko, 2016) proposes an extension to such an economic approach for time-varying demand for parking but keeps a rather limited description of traffic congestion. (Benenson *et al.*, 2008) and (Levy *et al.*, 2013) elaborate agent-based models for parking in cities. The description of the search process is very detailed but the impacts of the surrounding traffic are poorly considered. (Gallo *et al.*, 2011; Boyles *et al.*, 2015; Xiao *et al.*, 2016; Du and Gong, 2016) investigate how parking search can be incorporated into static and dynamic traffic assignment models by refining the cost functions. The dynamic and mutual interactions between traffic conditions and parking searches are not fully considered. Reservoir models based on the concept of the Macroscopic Fundamental Diagram (MFD) (Daganzo, 2007; Daganzo and Geroliminis, 2008) are very appealing as they provide fast implementations of a dynamic traffic simulation at large scales. (Geroliminis, 2015; Liu and Geroliminis, 2016) propose an extension of such approaches to characterize the effects of parking on traffic operations during the peak hour in congested cities. (Amer and Chow, 2016) suggest a quite similar framework but include an extension to account for delivery demand and delivery trucks that double park. Finally, (Cao and Menendez, 2015) certainly presents the most effective and detailed representation of the interactions between parking searches and traffic congestion in a dynamic manner. It notably accounts for the competition between vehicles when available parking spots are rare. However, as the modelling framework is time-based, the authors have to assume that the parking process is globally renewed at the end of each time slice.

In this paper, we propose to pursue the effort in (Geroliminis, 2015; Cao and Menendez, 2015; Liu and Geroliminis, 2016) to develop a simple and dynamic model able to assess the mutual influence of on-street parking and traffic conditions at large scale. Here, we will resort to a trip-based formulation of the MFD reservoir dynamics firstly proposed in (Arnott and Rowse, 1999; Arnott, 2013; Lamotte and Geroliminis, 2016) and whose properties are analytically investigated in (Mariotte *et al.*, 2017). This formulation permits to keep track of all individual vehicles in a given area (reservoir) while the mean spatial speed is given by the MFD with respect to the current accumulation. This formulation makes it possible to individualize the trip-length of all vehicles and then to precisely consider the

parking search phase depending on the parking occupancy, i.e. the ratio between the number of occupied and free parking spots. With regards to this latter process, experimental data from the city of Lyon (France) are analyzed to refine parking search laws previously proposed in the literature. In particular, we investigate the relationship between the distance to park and the parking occupancy. We chose to investigate the distance travel instead of the travel time because the former is less sensitive to the traffic mean speed. Numerical investigations are also conducted to further understand how the distance to park evolves when the parking occupancy dynamically changes. Finally, the trip-based MFD and the distance to park models are combined to demonstrate the capabilities of the full framework. Three different scenarios are investigated. The first two correspond to strategies that switch the demand from on- to off-street parking. While being very classical, these scenarios aim to show that the model properly reproduces the influence of on-street parking searches on traffic conditions by alleviating congestion when more vehicles go to off-street parking facilities. The third scenario assesses by simulation the effect of introducing a smart-parking technology, i.e. an embedded application able to inform drivers when a parking spot is available on one of downstream links of the next intersections. These three scenarios are contrasted and highlight how versatile the proposed framework is.

The paper is organized as followed. Section 1 investigates the relation between the travel distance and the parking occupancy. Section 2 presents the trip-based model and its extension to account for on-street parking searches. Section 3 presents the simulation results for the three scenarios. Section 4 is the conclusion.

1. On-street parking search process for a single vehicle

In this section, we focus on the on-street parking search process. Let denote τ the parking occupancy, T and D respectively the mean time and distance to park. We first propose a short literature review of the relation between these variables based on empirical or analytical analysis. We then extend one existing relationship to better reproduce new experimental findings for low to medium τ values. Finally, we numerically investigate what happens when τ experiences fast time variations.

1.1. Distance or time to park formulae: a literature review

Most of existing empirical studies consist in surveys based on on-board vehicle measurements or driver interviews to collect the experimented time or distance to park. This provides a qualitative vision of the easiness to park in a given area or city. Manual counts or data fusion from different sources (ticketing, floating cars...), e.g. (Cats *et al.*, 2016; Thornton *et al.*, 2014), permit to estimate τ . Unfortunately, studies where both T (or D) and τ are available are quite rare. In 1985, (May and Turvey, 1985) realized a survey in Mayfair, London (UK) and proposed a decreasing exponential expression for the relation between T and τ .

$$T = ae^{-b\tau} \quad \text{Eq. (1)}$$

In 1994, (Axhausen *et al.*, 1994) chose the following parabolic relation between the same variables based on surveys conducted in Frankfurt. However, this study had for primary purpose off-street parking.

$$T = \frac{a}{1-\tau} \quad \text{Eq. (2)}$$

Most of the few other recent empirical studies simply calibrate one of these two functions on new observations, e.g. (Shoup, 2006) and (Belloche, 2015). Surprisingly, all the studies are based on T although it is not only sensitive to τ but also to the mean network speed. Using D instead of T permits to eliminate this latter dependency. This is done in studies based on probabilistic analysis. (Arnott and Rowse, 1999) develops an analytical model for parking. It assumes that the probability to find a vacant parking spot in a space interval dy is constant and equal to pdy . Thus, the probability for a driver to find its first free spot after driving a distance y is defined by $pe^{py}dy$. (Geroliminis, 2015) elaborates the discrete counterpart of this model that consists in screening successively all spots until finding a free

one. Each screening operation corresponds to a Bernoulli trial with a probability of success p defined as the ratio between the number of free and total parking spots, i.e. $p=1-\tau$. Let N_1 be the rank of the first free spot. N_1 follows a geometric distribution when all screening operations are independent.

$$P(N_1 = i) = (1 - p)^{i-1} p \quad \text{Eq. (3)}$$

Thus, $E(N_1)$ is equal to $1/p$, where $E(\cdot)$ is the expectation (mean) function. Let denote l_s the mean distance between two successive parking spots. It comes that:

$$D = \frac{l_s}{p} = \frac{l_s}{1 - \tau} \quad \text{Eq. (4)}$$

Note that Eq. (4) and (2) look similar. However, as T and D are related through the mean speed over the network, the parameter a in Eq. (2) should certainly be calibrated again when traffic conditions change.

As the discrete screening process seems very reasonable, it seems that Eq. (4) or (2) should be preferred to exponential expressions like Eq. (1) when calibrating the relation between T (or D) and τ . (Bilfulco, 1993) presents a similar discrete process but assumes that draws are done without replacements. This means that p increases with the number of draws because the first draws result from occupied spots. It is well-know that draws with or without replacements lead to close results when the number of spots is high, which is usually the case for parking. Note that (Bilfulco, 1993) also propose an expression for D when τ varies with time. Unfortunately, this expression is no longer relevant for fast variations of τ as it is assumed that such variations are slow enough for the geometric distribution to remain valid over time periods with constant τ . Let $f(\tau)$ be the distribution of τ over the global time period, the expression of D becomes:

$$D = \int E(D|\tau) f(\tau) d\tau = l_s \int \frac{f(\tau) d\tau}{1 - \tau} \quad \text{Eq. (5)}$$

1.2. Distance to park with constant parking occupancy

From the literature review, it appears that Eq. (4) is the most relevant. We now try to calibrate this relation further referred as SP1 on real data kindly provided by the CEREMA in France. Such data has been collected in ten different districts of the city of Lyon/Villeurbanne (France) in 2008. For each district, several random destination addresses are assigned to some drivers. Each driver has first to reach the address and then starts looking for a parking spot in a pre-defined area that permits to walk no more than 250 m to finally reach the destination. Time and distance to park are recorded. Simultaneously, other investigators walk through the different regions to estimation the value of τ . The global experiment lasted four days and τ was estimated on average for three different time periods every day: morning (9h30-11h45); afternoon (14h15-16h30) and evening (17h-20h). Further details about the experimentation can be found in (Belloche, 2015).

The data for six districts over ten were unusable because either the range of the τ observations was too small or too few replications were made. Figure 1d-f show the calibration results for SP1 and the four remaining areas. Figure 1b synthesizes the results and provides two goodness of fit indices: the r -square (r^2) and the root mean square error (RMSE). Goodness of fit is good when τ values are high, i.e. for areas 1 and 3 but is very poor when low to medium values of τ is observed (below 0.9) like in area 2 and 4. Furthermore, the fitted values for l_s appear inconsistent in most cases as it should represent the distance between two successive parking spots. l_s is equal to 6.5 m for area 1 but to 46 m, 75 m and 44 m for the three remaining areas even when the goodness of fit is good.

We then decided to extend the search process SP1 to account for the distance travelled where no parking spot exits (intersection crossing, reserved places, urban furniture...). Figure 1a presents the sketch of the new process SP2 compared to SP1. Now, every bunch of m spots, drivers have to cover a distance l_{ms} without any chance to find a place. This alternation is repeated until the driver finds a free spot within the repetition of the Bernoulli trial over each bunch of spots. One sequence combining the no spot distance and a bunch of m parking spots roughly represents one typical link of the network

between two intersections. We further assume that the value of m is the same for all sequences and that the process is initiated by driving a first l_{ns} distance. Note that l_{ns} can easily be distributed but the first assumption is necessary to ensure analytical tractability.

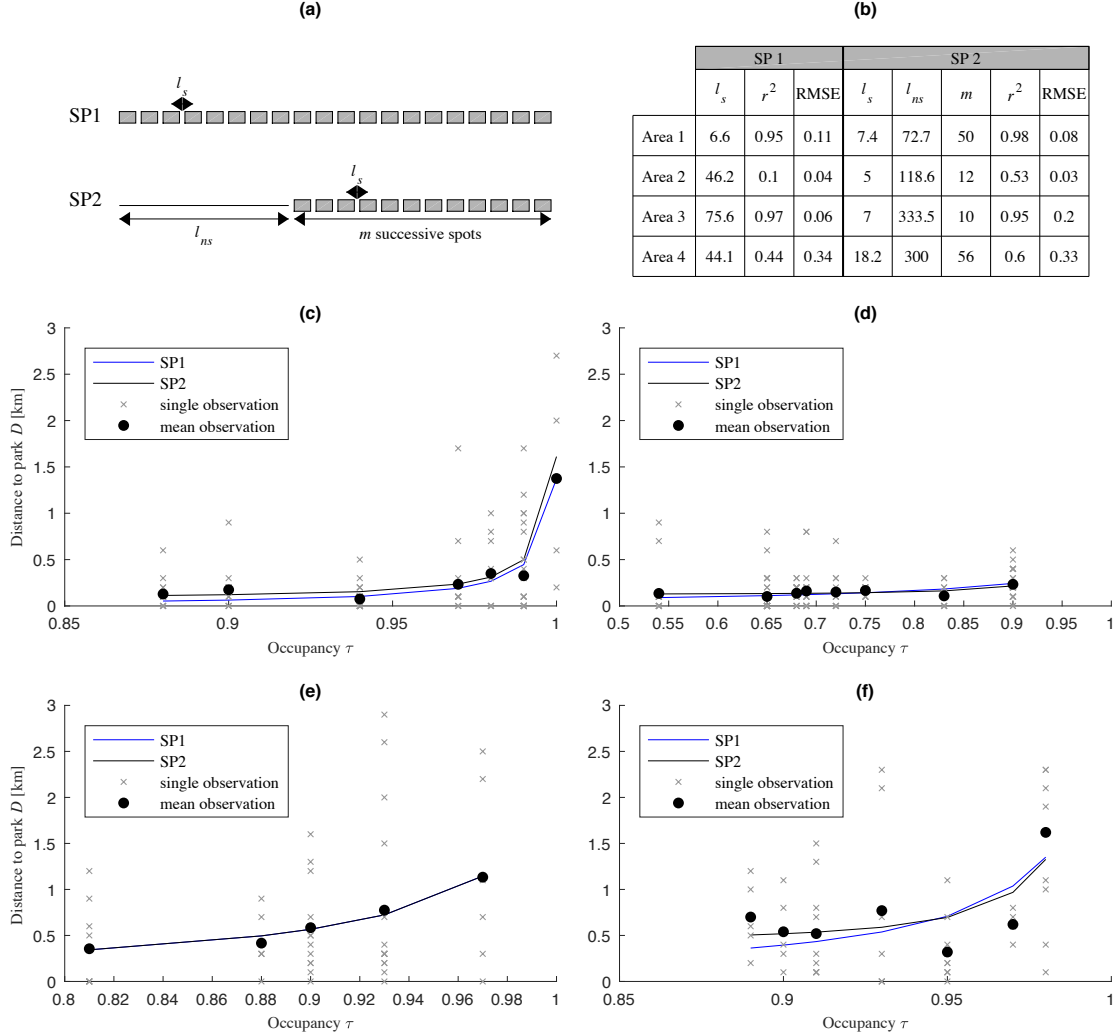


Figure 1: On-street parking search process (a) SP1 vs. SP2 parking processes (b) synthesis of calibration results (c) to (f) calibration results for areas 1 to 4.

The mean travel distance within bunches before finding a place is still given by Eq. (4). Let N_2 be the number of bunches that a driver has to screen. The travel distance excluding the bunches (where no spot exists) is equal to $N_2 l_{ns}$. Noticing that the probability that a driver screens a full bunch without finding a free spot is equal to τ^m , it comes that N_2 also follows a geometric distribution with a probability $p = 1 - \tau^m$. Finally, the mean distance travel for the full SP2 process is given by:

$$D = l_{ns} E(N_2) + l_s E(N_1) = \frac{l_{ns}}{1 - \tau^m} + \frac{l_s}{1 - \tau} \quad \text{Eq. (6)}$$

Figure 1d-f show the calibration results for SP2 and the four areas. Both SP2 and SP1 processes lead to very close results for high τ values ($\tau > 0.9$). SP2 process increases the mean travel distance for low to medium τ values, which appears more in line with the experimental observations even though data points are very sparse in such a range. Selecting the SP2 process significantly improves the goodness of fit indices compared to SP1 for areas 1, 2 and 4. Goodness of fit was already very good for SP1 and area 3. The most important benefit of introducing SP2 is that all parameter values are consistent with respect to their physical meaning in all cases on the contrary to SP1, see Figure 1b. In particular, the l_s values are now equal to 7.4, 5, 7 and 18.2 m.

Finally, the variance of D can also be determined for SP2 by noticing that the random variables N_1 and N_2 are not independent, as $N_2 = I(N_1/m)$ where I is the integer part:

$$V(D) = l_{ns}^2 V(N_2) + l_s^2 V(N_1) + 2l_{ns}l_s (E(N_1N_2) - E(N_1)E(N_2))$$

$$\text{where } V(N_1) = \frac{\tau}{(1-\tau)^2}; \quad V(N_2) = \frac{\tau^m}{(1-\tau^m)^2}; \quad E(N_1N_2) = \sum_i iI(i/m)\tau^{i-1}(1-\tau) \quad \text{Eq. (7)}$$

1.3. Distance to park with fast time-variant parking occupancy

In the previous subsection, we assumed that τ is constant. Slow time variations of τ can easily be handled by either (i) considering successive time periods with constant τ and applying Eq. (4) for SP1 or Eq. (6) for SP2 or (ii) applying Eq. (5) that can be straightforwardly extended to SP2 considering the distribution of the observed τ values over a global time period. In this subsection, we investigate the case when τ experiments fast variations. In this latter case, the probability p of successfully finding a free spot is changing for each Bernoulli trial.

This part of the study is based on numerical investigation of the successive trials corresponding to SP2. In practice, we define an initial value for τ , i.e. τ_{ini} and increases or decreases it by a value of 0.005 every time step Δt . We generate 1000 vehicles that have to park at time $t=0$ and apply SP2 process by modifying the value of p every Δt when drivers are exploring a bunch of existing spots. When a Bernoulli trial is successful for a given vehicle, we stop the process and record the travel distance. As the variations of p are defined in time, we use a constant travel speed V_m to determine the number of places that can be screened during Δt . Finally, we calculate τ_m the mean τ value over the period required for 95% of the vehicles to park.

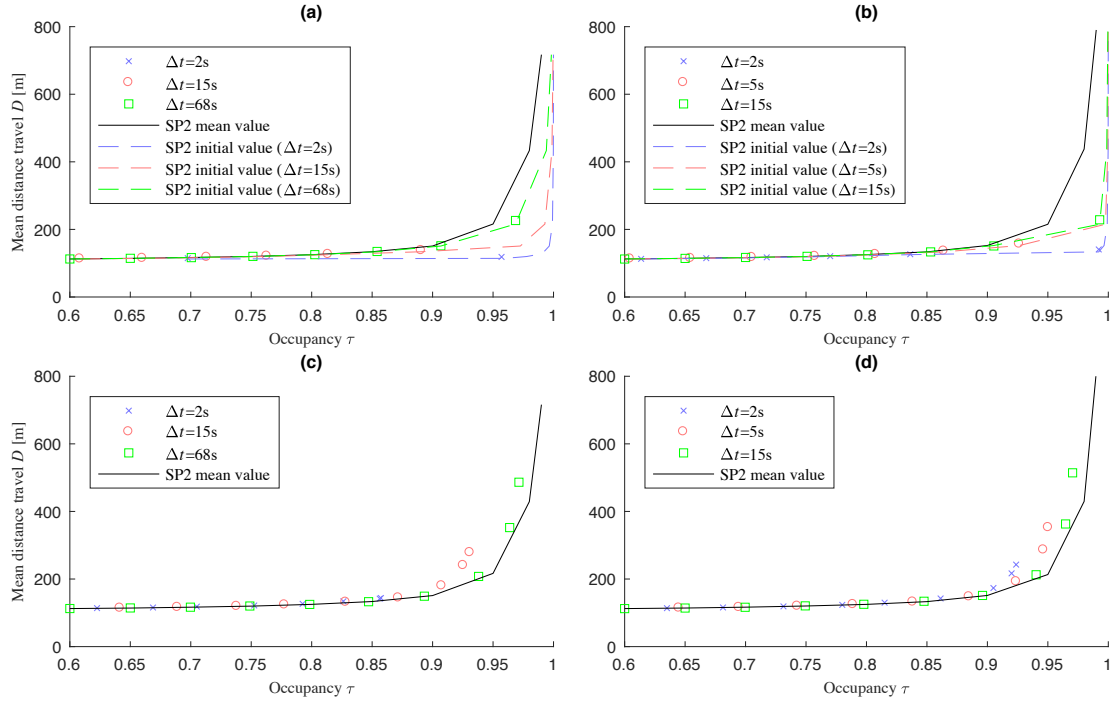


Figure 2: Numerical investigations of SP2 with fast time-variations of τ (a) Increasing τ - $V_m=2.8$ m/s (b) Increasing τ - $V_m=15$ m/s (c) Decreasing τ - $V_m=2.8$ m/s (d) Decreasing τ - $V_m=15$ m/s

Figure 2 shows the mean travel distance with respect to τ_m for increasing and decreasing τ step-profiles, two V_m values (2.8 m/s and 15 m/s) and three Δt values (2, 5 and 68 s). Figure 2a&b correspond to increasing τ profile. The numerical results are compared to Eq. (6) apply with either $\tau=\tau_m$ or $\tau=\tau_{ini}$. It appears that whatever Δt values, the numerical values can be closely estimated with Eq. (6) and $\tau=\tau_{ini}$. This means that τ_{ini} is predominant when τ increases during the repetition of the Bernoulli trials. This can be explained by the fact that when τ_{ini} is low to medium, drivers quickly find

a spot and thus the initial state is the most important. When τ_{ini} is high the increase of τ makes even harder to park with time. So again, the most important part of the process is the beginning. Note that we disregarded situations where τ may reach 1 as drivers can no longer park. Figure 2c&d correspond to decreasing τ step-profile. In that latter case, it appears that Eq. (6) and $\tau = \tau_m$ provides a fair estimation of the numerical results. τ_{ini} is no longer the key variable here because drivers can take benefit of reduced τ with time to more easily find a free spot.

This analysis is of course very preliminary but it provides rough guidance on how to apply Eq. (6) over a time period when τ quickly evolves. When τ is increasing, τ_{ini} should be preferred when estimating the mean travel distance to park. When τ is decreasing, τ_m is more relevant.

2. Trip-based macroscopic models including parking search

In this section, we first introduce a trip-based dynamic macroscopic model able to reproduce the time-evolution of traffic conditions in a given area while considering different user categories depending on their parking strategy. Second, we implement in this framework the search process SP2 for users that are looking for on-street parking, while dynamically monitoring in parallel the parking occupancy. Such a combination leads to a simple but efficient global framework that can finely reproduce the interactions between the parking search process and the dynamic traffic conditions. Simulation results for three different scenarios will be presented in the next section.

2.1. Basics of the trip-based MFD model

Let consider the urban network included in a given spatial area. This area is represented by a single reservoir whose accumulation (number of vehicles) $n(t)$ varies with time. Traffic dynamics within the reservoir is characterized by a speed-MFD $V(n)$ and a mean trip length L . Traditional (accumulation-based) MFD simulation approaches use the following conservation equation to describe evolution of $n(t)$:

$$\frac{dn(t)}{dt} = Q_{in}(t) - Q_{out}(t) \quad \text{Eq. (8)}$$

where $Q_{in}(t)$ and $Q_{out}(t)$ are respectively the inflow (demand) and the outflow. The outflow can be derived from the speed-MFD, i.e. $Q_{out}(t) = nV(n)/L$. This formulation leads to inconsistent wave propagation within the reservoir when the inflow varies quickly or significantly, like for large step-functions, see (Leclercq *et al.*, 2015; Lamotte and Geroliminis, 2016; Mariotte *et al.*, 2017). This problem is the same as what is observed for exit-flow models (Merchant and Nemhauser, 1978; Friesz *et al.*, 1989) and classical link flow theory: the outflow reacts instantaneously to inflow variations whatever the travel distance is. Furthermore, considering different trip length for vehicles is difficult (Mariotte and Leclercq, 2016) as it requires to cluster vehicles into groups with homogeneous trip lengths and to define a conservation equation per group leading to a system of equations (Yildirimoglu *et al.*, 2015).

To circumvent these drawbacks, a new formulation has been proposed in (Lamotte and Geroliminis, 2016) following first attempts in (Arnott and Rowse, 1999) and (Arnott, 2013). Let $N(t)$ be the vehicle that enters the reservoir at time t . $N(t)$ is defined continuously as the integral of the total inflow (cumulative vehicle function). The travel time T of vehicle $N(t)$ is determined by integrating the mean reservoir speed until the vehicle has reached its targeted travel distance L :

$$\int_t^{t+T(N(t))} V(n(u)) du = L \quad \text{Eq. (9)}$$

This model will be further referred as trip-based MFD formulation as the traffic conditions in the reservoir are still characterized by the speed-MFD while the model now explicitly focuses on each vehicle trip within the reservoir. Note that the accumulation is now only implicitly defined compared to the accumulation-based MFD formulation. The simplest way to determine $n(t)$ is to calculate the difference between $N(t)$ and $N_{out}(t)$, where $N_{out}(t)$ is the cumulative function for trip endings.

A fundamental difference between the trip-based and the accumulation-based models is that the former explicitly considers the delay (travel-time) between the inflow and outflow boundaries. (Arnott, 2013) and (Mariotte *et al.*, 2017) show that Eq. (9) can be rephrased to express the relationship between Q_{in} and Q_{out} , see Eq. (10). The endogenous delay term, i.e. the vehicle travel time T , plays a crucial role when relating the in- and outflows. The accumulation-based model does not account for any delays between the in- and outflows, see Eq. (10). This explains the potential drawbacks mentioned above for this latter model.

$$Q_{out}(t) = Q_{in}(t - T(N(t - T))) \frac{V(n(t))}{V(n(t - T(N(t - T))))} \quad (\text{trip-based model})$$

$$Q_{out}(t) = Q_{in}(t) + \frac{dn(t)}{dt} \quad (\text{accumulation-based model}) \quad \text{Eq. (10)}$$

Interestingly, Eq. (9) can be straightforwardly extended to individualize vehicle trip lengths, which is not the case for the accumulation-based model (Aboudolas and Geroliminis, 2013; Ramezani *et al.*, 2015). A specific travel distance $L(N(t))$ can be assigned to each entering vehicle $N(t)$. This travel distance can include a specific part related to the parking search process, which makes the trip-based formulation very relevant for simulating parking operations. Note that this travel distance does not need to be fully defined when a vehicle is entering. So, the distance to park can be dynamically updated depending on the evolution of the parking occupancy.

(Lamotte and Geroliminis, 2016) proposes a numerical method to solve Eq. (9) when the inflow is discretized into vehicles. Each vehicle in the network drives a distance $V(n)\delta t$ every time-step δt until it has reached its targeted travel distance. N_{out} is incremented by 1 every time a vehicle finalizes its trip. Here, we resort to a similar but more computationally efficient method, i.e. a fully event-based scheme. Such algorithm has been first introduced in (Mariotte *et al.*, 2017) but in the simplest case where trip-lengths are predefined before entering the reservoir. In the next subsection, we describe how the event-based scheme has been extended in this paper to (i) account for dynamic updates of the vehicle trip-length due to the parking search processes (ii) consider different user classes depending on their parking strategies and (iii) dynamically interact with the parking occupancy.

2.2. Numerical implementation of the trip-based MFD model with on-street parking search

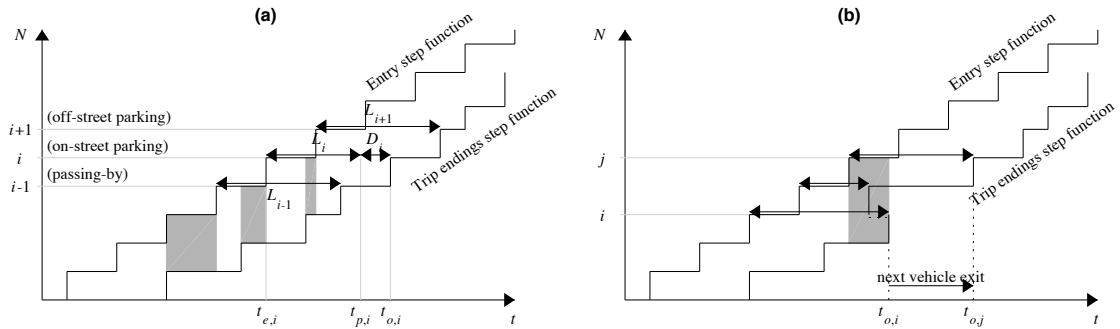


Figure 3: Event-based resolution of the trip-based MFD model (a) general principle (b) FIFO exit times with respect to the original exit time order

Figure 3a describes the functioning of the event-based numerical scheme including the on-street parking search. The inflow is first discretized into vehicle unit. Three states are defined for a vehicle: ‘starting’, ‘searching’ and ‘ending’. Events happen every time a vehicle changes its state. ‘Starting’ state is assigned to vehicles that either enter the reservoir (external demand) or that start their trip within the reservoir (internal demand). External demand vehicles can either end trips within the reservoir or just pass by. ‘Searching’ state is only assigned to vehicles that look for on-street parking. Other vehicles that end their trips within the reservoir (off-street parking) disappear from the network as soon as they have covered their travel distances $L(N)$. ‘Ending’ state is assigned to all vehicles whatever their destination is as soon their trip length is covered. On-street parking vehicles should not only drive their travel distance but also an extra one corresponding to the parking search process SP2,

see vehicle i in Figure 3a. To sum up a vehicle can belong one of the following five categories, see Figure 4a:

- In→out: Passing-by (external demand → external destination)
- In→off: Off-street parking (external demand → internal destination)
- In→on: On-street parking (external demand → internal destination)
- Off→out: Off-street internal departure (internal demand → external destination)
- On→out: On-street internal departure (internal demand → external destination)

All vehicles experience the ‘starting’ and ‘ending’ states. Only on-street parking vehicles are assigned the extra state ‘searching’ that triggers the calculation of the distance-to-park, see below. A different travel distance L is assigned to all these five categories. Note that such distances can even be randomly distributed following a given distribution as the modelling framework permits to individualize the distance travelled by each vehicle.

Starting times are defined by the demand scenario. Note that external demand vehicles can be delayed outside the reservoir as the total inflow is bounded by the reservoir supply function defined by the MFD. Between two events, the number of vehicles within the reservoir n is constant and so is the mean speed given by the MFD $V(n)$. This speed is assigned to all current vehicles that has not yet reached an ‘ending’ state. All vehicles in the reservoir are then driving a common distance equal to $V(n)\Delta t$, where Δt is the time that separate two successive events. Ending times are calculated for all vehicles within the reservoir and each event by applying Eq. (9) considering the remaining distance to travel, i.e. the travel distance L minus the distance already travelled in the reservoir since the vehicle entered.

In practice, every time an event occurs, the accumulation n is updated as so the mean speed V . For all vehicles within the reservoir the distance already covered within the reservoir is also updated as well as the ending time assuming V will remain constant. Thus, each vehicle move forward into the reservoir step-by-steps with the right mean speed and the list of the next ending times is properly updated. This is important to determine the next events among the starting and ending times. Note that the next event will trigger an update of n and V , and so of all covered distances and ending times.

Note that this numerical implementation is simple but can be further improved to reduce the calculation burden. For now, covered distances and ending times are updated for all vehicles at every event. When n is high, this can lead to a high load of updates. However, as all vehicles in the reservoir experiment the same speed between two events, the order of trip endings with respect to time only depends on the starting time and the travel length of each vehicle. So, it is not necessary to update the ending times for all vehicles at each event but only the ending time of the next vehicle that will end its trip. The ending times for other vehicles can be later adjusted based on the common record of experimented speeds when it is the turn of a particular vehicle to end its trip. An example is provided in Figure 3b. Vehicle j is the next vehicle to end its trip after vehicle i . Thus, only the ending time of vehicle j has to be updated when vehicle i ends its trip based on the different speed values that vehicle j has experienced during its travel.

We still have to address the particular case of on-street parking vehicles. Such vehicles reach the ‘searching’ state after completing their initial assigned distance $L_{\text{In} \rightarrow \text{on}}$. At this moment, their distance to park D is determined on an individual basis based on the SP2 process, see section 1. When this distance is covered an ‘ending’ event happens. Note that the ending times of such vehicles should be updated at every event as their travel distance evolves during the search process.

The parking occupancy τ for the reservoir is updated either when an on-street parking vehicles reaches an ‘ending’ or when on-street internal departure ‘starting’ event occurs. So, τ dynamically evolves during the simulation. Let denote n_{tot} the total number of parking spots and τ_0 the parking occupancy at the beginning of the simulation. To calculate D , we resort to Eq. (6). Every time τ changes, the distance to park is refreshed for all current searching vehicles based on the new τ value. The distance already covered while searching for a parking spot is deduced from the new D values. Because Eq. (6) is theoretically only valid if τ is constant during the full search process, we use in Eq. (6) the mean

value of τ over a rolling horizon corresponding of the last five parking events instead of its instantaneous value. Note that Eq. (6) can be replaced at a higher computational cost by implementing the full Bernoulli trial replication corresponding to SP2. Because the geometrical distribution is memoryless it is possible to perform the process until the rank of the next free spot is determined for a given τ and perform it again when τ is updated keeping only the distance already covered. This last solution permits to capture the full distribution of the distance to park and not only the mean value.

2.3. Consistency between the trip-based MFD model and the parking search process

The parking search processes (SP1 or SP2) and the dynamic traffic model (trip-based MFD) are based on two different abstractions of the same real network. SP2 process is founded on the notion of typical link as described in section 1. This means that during the parking search, vehicles are assumed to follow a random path into a simplified network corresponding to a succession of typical links with the same amount of parking spots. However, as described in the previous subsection, we do not handle parking spots either on an individual basis or at the link level. Only a global parking occupancy τ is considered for the full region. This is consistent with how the parking process has been represented, i.e. a nested geometrical distribution with a common probability p for a spot to be free, see Eq. (6).

Dynamic traffic conditions are characterized by an MFD model where the speed is homogeneous among the reservoir and only depends on the current vehicle accumulation. The real network is synthetized both into the MFD shape and the definition of the travel distances by vehicle category. This means that the link speed experimented by vehicles during the search process only depends on the total accumulation in the reservoir and not on potential local events. In particular, we do not account for speed reductions during parking maneuvers or local heterogeneities in parking occupancy. This is consistent with the way we represent the parking search as local heterogeneities are not considered except as a common and global geometrical process.

Considering such heterogeneities would certainly require microscopic simulations, which are not comparable with the proposed approach in terms of complexity and calibration burden. Here, we are focusing on a simple and aggregate approach, which circumvent the current limitations of similar existing approaches mentioned in the introduction. The model we have designed and presented in the two previous subsections is able to (i) reproduce the dynamic interactions between the parking and reservoir loads (parking occupancy and mean vehicle speed) for all vehicle categories and not only for parkers, (ii) individualize travel distances in particular for on-street parkers, which was not the case in previous attempt, e.g. (Geroliminis, 2015) and (iii) provide a longitudinal (trip-based) description of the parking search process.

Such properties are going to be highlighted in the next section. Finally, it should be noticed that this paper focuses on the description of the traffic/parking dynamics within a reservoir but this framework can easily be extended to deal with multiple reservoirs in the spirit of (Aboudolas and Geroliminis, 2013; Ramezani *et al.*, 2015). This is a promising solution to handle heterogeneous traffic/parking conditions by focusing on small areas that can be considered as homogeneous.

3. Simulation results for two different scenarios

This last section aims to illustrate the capabilities of the trip-based MFD model with parking search. First, simulation results are presented for a reference scenario mimicking a 3*3 Manhattan network. Then, three different parking strategies are tested. First, a switch of the demand to park is operated from on- to off-street parking to verify that the model properly reproduces the expected improvement of the traffic conditions. Second, such a switching process is made reactive to the on-street parking occupancy and the network traffic conditions to mimic the outputs of a parking choice model, e.g. (Waraich and Axhausen, 2012; Horni *et al.*, 2013). Third, the model is used to assess the impacts of the deployment of a smart parking technology. Such an application informs drivers crossing an intersection if a free spot is available on one of the downstream links.

3.1. Definition of the reference scenario

We consider a reservoir including a total network length equal to 3000 m. The travel distance is 500 m for passing-by vehicles (In→out) and 250 m for all other vehicle categories (In→off, In→on, On→out, Off→out), see section 2 for the category definition. The trip-based MFD model only uses these values as a proxy for the real network without requiring a complete description of the underlying topology, see Figure 4a. To implement the SP2 process for on-street parking, we need to refine the description of the network by parametrizing the notion of typical link, see section 1. Here, we assume that the network can be divided into 24 typical links of 125m: $l_{ns}=50$ m, $l_s=5$ m and $m=15$. As such, the network has 360 available parking spots covering 60% of the network total length. This setting mimics for example a 3*3 Manhattan network, see Figure 4b, but can adjust many more network configurations.

Figure 4c presents the MFD in speed that is piecewise linear in flow. Figure 4d defines the time-evolution of demand inflows for all five vehicle categories. This represents what usually happens during a three-hour peak period with a global demand increase and then decrease. Note that the fractions of passing-by (In→out), off-street parking (In→off) and off-street internal departure (Off→out) remain constant during the whole period and are respectively equal to 50%, 5% and 5%. The fraction of on-street parking vehicles (In→on) is equal to 21% while the fraction of on-street internal departure (On→out) is equal to 19% during the first 83 min period. During this period τ is going to increase until parking spots become very rare, i.e. when τ is close to 1. Then, the two fractions respectively switch for the second-time period to 19% and 21% to reduce τ . Note that initial on-street parking occupancy τ_0 at time $t=0$ is set to 0.42. Off-street parking is assumed to have an infinite capacity in this study.

Finally, it is important to mention that we do not explicitly account for parking duration in this framework. However, this phenomenon is implicitly considered when setting-up the departure rate (demand) from off- and on-street parking. The framework can easily be extended by considering that on-street parking departure times are no longer exogenously defined but depend on arrival times for vehicles that parked.

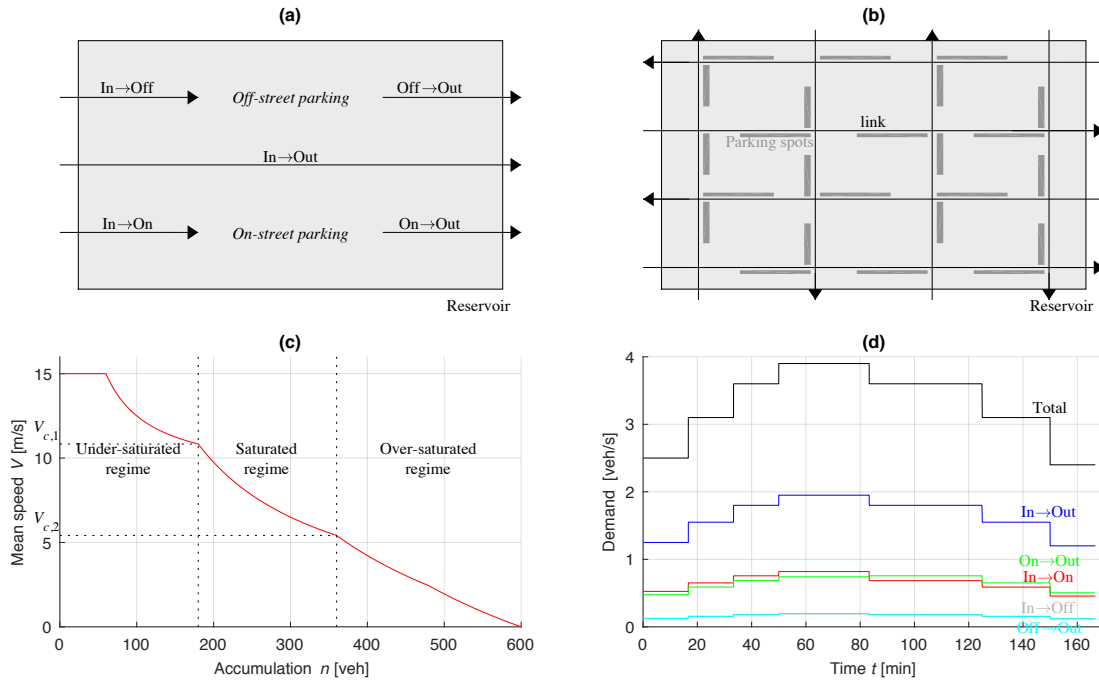


Figure 4: Simulation setting for the reference scenario (a) Definition of vehicle categories (b) Network setting for the SP2 parking process (c) MFD setting in speed (d) Demand setting for all vehicle categories

3.2. Simulation results for the reference scenario

Figure 5 presents the simulation results for the reference scenario. Figure 5a shows the time-evolution of the accumulation n_i for all five vehicle categories and the total accumulation n . When $t < 61$ min, the accumulation values simply adjust the progressive increases of the demands. The global traffic state

remains in the under-saturated regime. This is confirmed by the time-evolution of the mean speed, see Figure 5c.

A major event happens at time $t=61$ min: the parking occupancy comes close to 1, see Figure 5b. From that instant, the accumulation for In→on vehicles starts quickly increasing as the distance to park becomes very high. The mean distance to park increases from 62 m before $t=61$ min to 634 m between $t=61$ and $t=108$ min. The accumulations for other vehicle categories follow the same trend with about a 3-min delay, see Figure 5a. This is because the mean speed into the reservoir is significantly reduced due to the accumulation of vehicles looking for parking spots, see Figure 5c. The model properly reproduces here two key phenomena related to parking: (i) the significant increase of the search distance when parking spots becomes rare and (ii) the impacts on the global traffic conditions through the reduction of the mean network speed.

At time $t=83$ min, the total demand starts decreasing while the fraction for departing vehicles from on-street parking slightly increases. This latter event means that more vehicles are willing to exit on-street parking than new vehicles are looking for parking spots. However, the parking occupancy remains close to 1 until time $t=108$ min. This time is needed to empty the pile of vehicles that have started their search. After time $t=108$ min, the parking occupancy quickly decreases. This phenomenon combined with the demand decreases make the accumulation drop for all vehicle categories. The accumulation values then stabilize to a free-flow stationary situation when all the vehicle stocks created during congestion have disappeared.

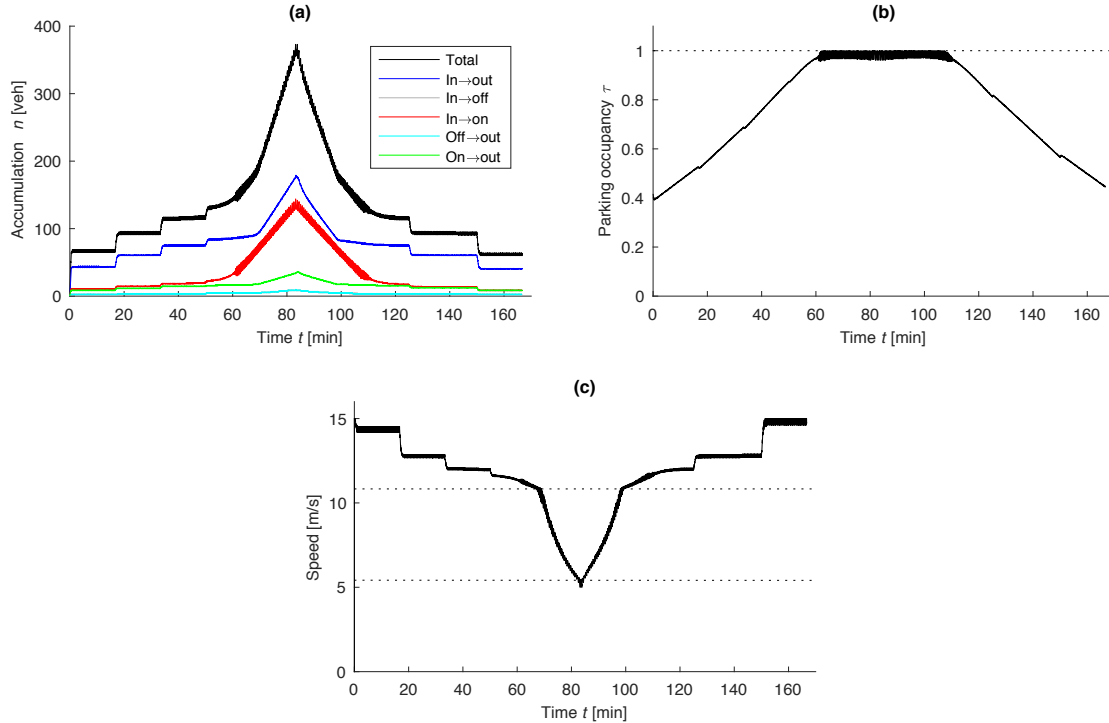


Figure 5: Simulation results for the reference scenario (a) time-evolution of the accumulation per vehicle category (b) time-evolution of the on-street parking occupancy (c) time-evolution of the mean network speed.

3.2. Switching the demand to off-street parking – static case

In the reference scenario, the ratio β between the mean demand for off- and on-street parking is equal to 0.25. In this subsection, we run simulations with exactly the same settings as the reference scenario except for the β value. By increasing β , we want to demonstrate that the model properly reproduces the congestion reduction we should expect for a switch from on- to off-street parking. In practice, such a switch can be obtained by pricing or incentive policies (increase of the price for on-street parking or discount prices for new users of parking facilities) or by increasing the off-street parking offer.

Figure 6 presents the simulation results. Figure 6a shows the time-evolution of the parking occupancy while Figure 6b shows the time-evolution of the mean network speed. Both figures highlight the same

trend: when β increases the saturation period for the on-street parking spots is reducing leading to higher network speeds and a shorter saturation period at the network level. When $\beta=1$, i.e. the demand for on- and off-street parking is similar, the saturation period even disappears both for the parking and the network.

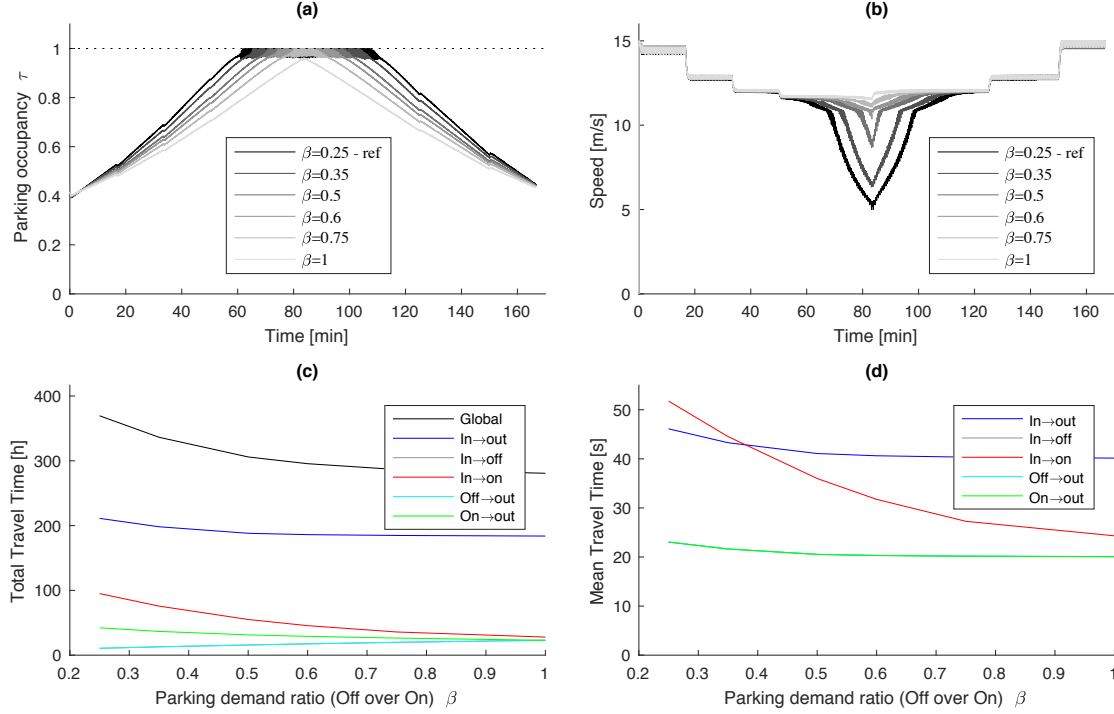


Figure 6: Simulation results when the demand to park is progressively switched to off-street parking (a) time-evolution of the parking occupancy for different β values (b) time-evolution of the mean network speed for different β values (c) Total travel times per vehicle category (d) Mean travel time per vehicle category

Figure 6c presents the total travel times for all vehicle categories and different β values. Most curves are decreasing when β is increasing except for the total travel times of In \rightarrow off and off \rightarrow out vehicles (these two curves are superimposed in Figure 6c). Switching vehicles to off-street parking improves the traffic conditions and thus the total travel time for most users except those which are using off-street parking facilities because they are more numerous. The most interesting result in Figure 6c is the evolution of the total travel time for all vehicles. It reduces from 369 h when $\beta=0.25$ to 295 h when $\beta=0.6$ (-20%). When β is further increasing, the reduction of the total travel time becomes negligible. This means that the model can be used to design optimal parking strategies. Here, the optimal balance between off- and on-street parking is close to 0.6 with respect to the network efficiency (total travel time for all users).

Finally, Figure 6d shows the mean travel time for all vehicle categories and different β values. In \rightarrow off, off \rightarrow out and On \rightarrow out curves are superimposed as they are related to the same travel distance (the speed in the reservoir is identical whatever the vehicle category is). This last figure highlights the improvements for all vehicle categories when β increases from 0.25 to 0.6. Not surprisingly, vehicles that have to park on-street take the most benefit of the demand switch to off-street parking. Note that the mean travel distance to park during the initial parking saturation period ($61 < t < 108$ min) reduces from 634 m when $\beta=0.25$ to 306 m when $\beta=0.6$ and to 78 m when $\beta=1$.

3.3. Switching the demand to off-street parking – reactive case

In the previous subsection, we simply consider different static values for the β ratio. In this subsection, we aim to highlight that the framework we proposed can easily handle more complex parking strategies and in particular reactive ones. Most parking simulation models, e.g. (Benenson *et al.*, 2008; Waraich and Axhausen, 2012; Honi *et al.*, 2013) or parking studies, e.g. (Axhausen *et al.*, 1994; Arnott and Inci, 2006) include a parking choice model to dynamically determine the ratio β with

respect to the parking occupancies, the traffic conditions and the implemented pricing strategies for both on-street and off-street parking. In this paper, we only aim to demonstrate the proper behavior of our simulation framework on simple test cases, so we are only going to investigate two reactive parking strategies.

The first and the simplest one corresponds to a very common behavior for people who are looking for on-street parking spots: after a certain time (or distance) the person is dropping his/her search to switch to an off-street facility. Here, we consider that In→on vehicles are switching to the In→off category after driving about 30% of the total network distance, i.e. 900 m and that they have to drive an extra 100 m to reach the nearest off-street facility. The second strategy is designed to mimic the outputs of a parking choice models. The β ratio is now dynamically adjusted with respect to the on-street parking occupancy and the mean network speed following Eq. (11):

$$\beta(t) = \frac{(\beta_1 - \beta_0)}{(V_{c,2} - V_{c,1})^2} \frac{(V(t) - V_{c,1})^2}{(1 - \tau_{ref})^2} (\tau(t) - \tau_{ref})^2 + \beta_0 \quad \text{if } V(t) \leq V_{c,1} \text{ and } \tau(t) > \tau_{ref} \quad \text{Eq. (11)}$$

$$\beta(t) = \beta_0 \quad \text{otherwise}$$

The rational for this equation is the following. When the speed corresponds to under-saturated traffic conditions, i.e. $V > V_{c,1}$ see Figure 4c, or when the parking occupancy is below the threshold τ_{ref} that characterizes parking saturation, β is equal to β_0 . β_0 is the beta ratio for the reference scenario, i.e. $\beta_0 = 0.25$, and τ_{ref} is fixed to 0.9. Otherwise, β follows a parabolic trend with respect both to V and τ . The derivatives of β with respect to V and τ are null respectively when $V = V_{c,1}$ and $\tau = \tau_{ref}$. We need a last point to fully define $\beta(t)$. We decide to fixe $\beta = 1$ (extremal case studied in the previous subsection) when $V = V_{c,2}$ and $\tau = 1$. $V_{c,2}$ is the speed when traffic conditions start being over-saturated, see Figure 4c. In practice, vehicles are now considered to belong to a single category when travelling their initial distance, i.e. 250 m. Then, a Bernoulli trial is performed to assign vehicles to either In→off or In→on categories depending on the current traffic and parking conditions.

Both strategies are respectively labelled sc1 and sc2 in Figure 7 that presents the simulation results. Figure 7a shows the time-evolution of the total and the In→on accumulations for both strategies and the reference scenario. For sc1, some vehicles start dropping their search after $t > 70$ min when the parking occupancy becomes fully saturated and the search distances have significantly increased. Such switches improve the traffic conditions, see Figure 7c for the mean speed evolution and also permit a faster on-street parking recovery after the top peak period, see Figure 7b for the parking occupancy evolution. The most interesting thing in this scenario is that only 44 vehicles in total switch from on-street to off-street parking. This corresponds to 0.7% of the total demand for on-street parking (2.2% of the peak period In→on demand between $60 < t < 100$ min) while 9% of the vehicles experiment a searching distance higher than the 900 m threshold in the reference scenario. This corresponds to a common observation, which is well reproduced by the simulation framework: switching vehicles not only benefit to all driving vehicles by improving traffic conditions but also to the other on-street searching vehicles. Figure 7d proposes an estimation of the dynamics of the β value. We define the outflow beta ratio as the ratio between the In→off and the In→on outflows calculating over a moving average of 8 trip ending events. The outflow beta ratio is only equal to β in free-flow conditions because the searching process for on-street parking significantly delays vehicle arrivals during saturation. Note that it is not possible to estimate directly β for sc1 and sc2 as some vehicles are changing their decision during their trip. It appears clearly in Figure 7d that when $60 < t < 70$ min, the outflow beta ratio is higher for sc1 than for the reference scenario because of vehicles that decide to move to off-street parking instead of on-street parking.

The second strategy (sc2) exhibits mostly the same trend. The switch from on-street to off-street parking appears to happen a few minutes sooner than for sc1 with a higher intensity, see Figure 7a&d. In fact, 55 vehicles switch in total in this scenario (compared to 44 in sc1), which represents 0.8% of the total demand or 2.5% of the peak hour demand. This is because vehicles are reacting faster to the decrease of the speed and the saturation of the on-street parking and may directly choose to switch

without starting a search. The speed evolution in Figure 7c again confirms that reducing parking searches during the peak hours significantly improves traffic conditions for all users.

Again, the objective here is not to study a detailed parking choice model but to show that the simulation framework we proposed can easily account for feedbacks on the parking demand and for competing strategies between on- and off-street parking. The simulation results provided in Figure 7 clearly show that the impacts of such feedbacks on traffic conditions for all vehicles and parking occupancy is well-reproduced by the model.

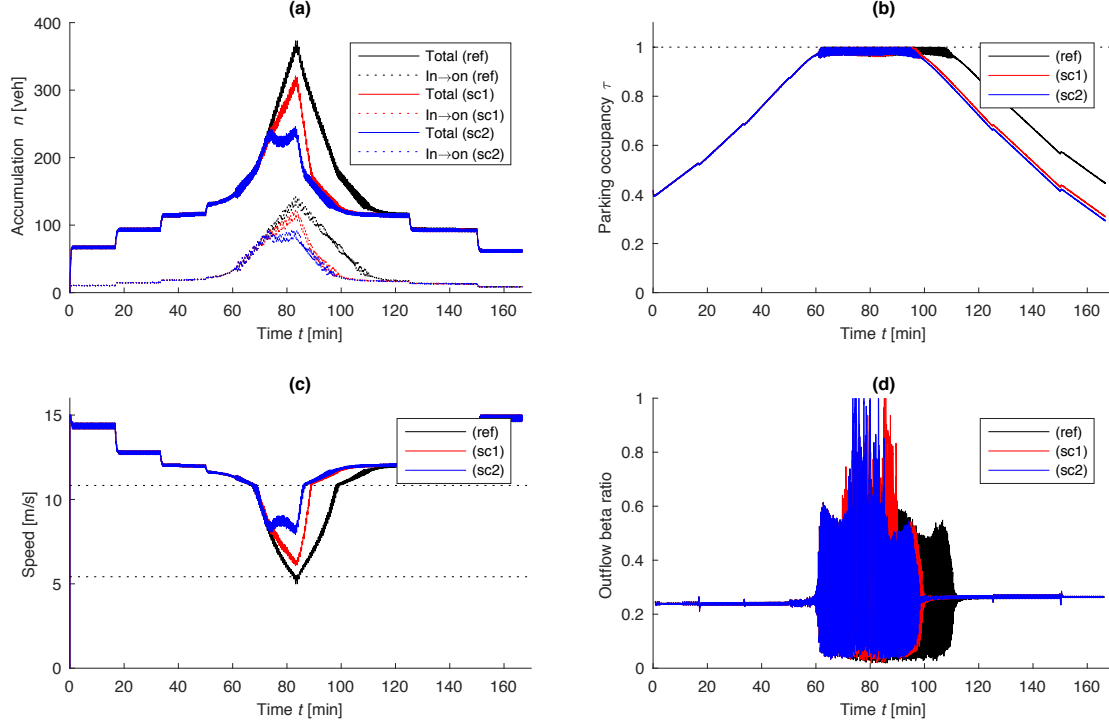


Figure 7: Simulation results for two reactive feedback strategies on the parking demand (a) time-evolution of the total and In→on accumulations (b) time-evolution of the on-street parking occupancy (c) time-evolution of the mean network speed (d) time-evolution of the outflow beta ratio.

3.4. Assessing the effect of a smart parking technology

The proposed framework can also quickly provide a global assessment when introducing new intelligent transportation systems. Here, we focus on an embedded technology able to inform drivers looking for parking at each intersection if an empty spot is available on one of the downstream links. We label such vehicles as smart versus the regular ones. The penetration rate is denoted r . Note that we are not considering how such a system may be deployed in the ground field because our primary purpose is to demonstrate that the simulation framework is not only able to assess the benefits for the smart vehicles but also the consequences for all user categories and in particular for the regular vehicles.

First, the SP2 process should be updated for smart vehicles. We assume that one sequence of the SP2 process, i.e. a l_{ns} distance followed by a bunch of m parking places – see Figure 1a, fairly well reproduces what happens for one typical network link. We further assume that drivers can choose among k downstream links at each intersection. The probability that none of these links has a free parking spot is equal to τ^{km} . In that case the driver has to drive a distance equal to $D_{link} = l_{ns} + ml_s$ before having a new chance that the application guides him/her to a free spot.

The search process SP2* for smart parking users is then the following: A Bernoulli trial with probability τ^{km} is performed. If successful, no parking spots is available and the vehicle should drive a distance D_{link} before a new trial can be operated (this generates a new event). In the other case, at least one parking spot is available and the application advises the right link. To finalize the vehicle trip, we should determine the mean travel distance D_f in the final link. This is given by:

$$D_f = l_{ns} + l_s \times E(N_1 | N_1 \leq m) = l_{ns} + l_s \frac{\sum_{i=1}^m i \times \tau^{i-1} (1-\tau)}{1-\tau^m} = l_{ns} + l_s \frac{m\tau^{m+1} - (m+1)\tau^m + 1}{(1-\tau)(1-\tau^m)} \quad \text{Eq. (12)}$$

When the distance D_f is covered, a final ‘ending’ event is generated.

SP2* has been implemented in the simulation framework. The only additional parameter is k , which is set to 3 for the simulation. Several penetration rates have been tested from 0% (the reference scenario) to 100% with an increment of 20%. Figure 8 presents the simulation results.

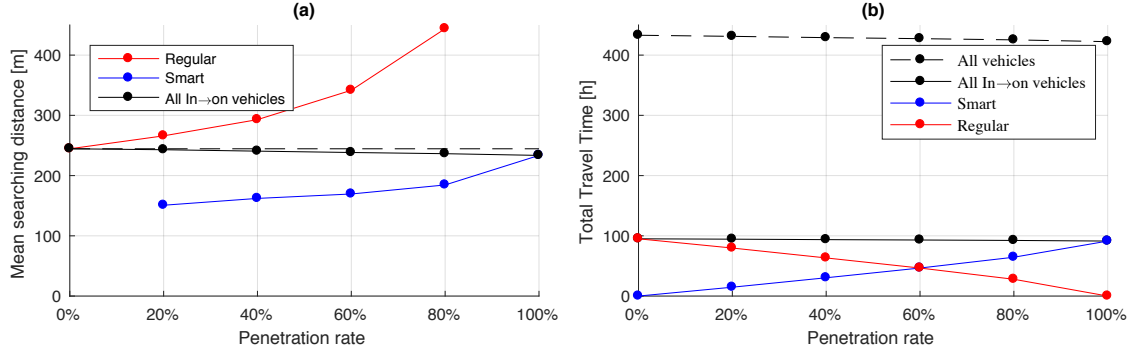


Figure 8: Simulation results considering the smart parking technology (a) Evolution of the total travel time with respect to the penetration rate (b) Evolution of the mean distance to park with respect to the penetration rate

Figure 8a shows the evolution of the mean searching distances during the full simulation period for smart and regular vehicles with respect to the penetration rate. It appears that smart vehicles have a significant gain over regular vehicles for low penetration rate. For $r=20\%$, the mean search distance for smart vehicles is 150 m compared to 265 m for regular users and a mean travel distance of 244 m when $r=0\%$. It is clear that the smart parking application highly favors the embedded vehicles at the expenses of the regular ones. This is particularly true when r is increasing to 60 and 80 %, see Figure 8b. However, the benefits for smart users is reduced when r increases as the value for information decreases when sharing in such a case. When $r=80\%$, regular drivers hardly find a spot as their mean travel distance increases up to 443 m. A noticeable result here is that when all users are equipped ($r=100\%$) the mean travel distance is lower by 4% (233 m vs. 244 m) than in the reference scenario ($r=0\%$). A more thorough analysis of the simulation results when $r=100\%$ compared to $r=0\%$ shows that searching distances are mainly reduced at the beginning and at the end of the top peak period. During the top peak ($60 < t < 100$ min), τ is almost equal to 1. Vehicles can hardly find a spot and the smart parking process does not provide significant improvements compared to a random walk. Even if we only test here a single parameter setting, these observations demonstrate that providing information is useful expect when parking is fully saturated as it permits to quicker fill the free spots.

Figure 8b presents the evolution of the total travel time with respect to the penetration rate. It confirms that the smart parking application leads to global benefit for all users. The total travel time is reduced by 2.5% when the penetration rate evolves from 0% to 100%. For In→on vehicles (looking for on-street parking), the reduction is even higher and about 4%. Note that the total travel time of smart users is increasing while the total travel time of regular users is decreasing because of the switch in vehicle number from regular to smart when the penetration rate increases.

Finally, introducing such a technology leads to global benefits for all users as it permits to reduce the mean global distance to park, which reduces congestion. However, the benefits come at the expense of regular users that experiment longer travel time and distance to park. Such conclusions are in line with the common sense. What is interesting here is that the proposed framework can precisely quantify the effects without requiring a high calibration and simulation burden.

4. Conclusion

This paper first investigates the relation between mean travel distance to park and the parking occupancy over an urban area. A new search process (SP2) based on the definition of a typical

network link including a distance without parking spots has been proposed. It appears to outperform existing formulations when analyzing experimental data in the city of Lyon/Villeurbanne. This search process has then been included in a trip-based macroscopic (MFD) model. This model permits to assign a different travel distance to each vehicle and thus to finely tune the behavior of the different driver categories inside a reservoir. Here, five categories have been distinguished depending on their parking strategy (on- or off-street parking), their origin (internal/external) and their destination (internal/external). An efficient event-based numerical scheme is used to solve the trip-based MFD model. This scheme has been extended to include a parking search component while monitoring the parking occupancy of the studied reservoir.

Simulation results show that the proposed framework perfectly reproduce the two key mutual interactions between on-street parking and traffic conditions. First, the saturation of parking spots increase the travel distances of searching vehicles. Second, these extra-distances induce major speed-drops when the network loading is high. So, traffic conditions worsen for the other vehicle categories. The reference scenario highlights particularly well such interactions because we observe both parking and network saturations during the demand peak. The capabilities of the proposed framework appear clearly when were studied three different parking strategies. When the demand is switched from on- to off-street parking, the congestion is alleviated. It is shown that the model can estimate the optimal ratio β between off- and on-street demand to guaranty the best network efficiency starting from the reference scenario. It is also shown that dynamic feedbacks on β can be easily implemented in the simulation framework, which paves the way for considering more detailed parking choice models. When a smart parking technology is introduced, the model is not only able to assess the benefits for smart users but also the impacts on the full population or a specific user category like the non-equipped vehicles. It should be pinpointed here that the proposed framework has few parameters that can be easily calibrated. Thus, it is possible to quickly have a first assessment of new parking policies or strategies without needing to perform extensive microscopic simulations.

The proposed modelling framework is very promising to quickly assess the performance of an urban area including parking operations. Note that we considered here that off-street parking vehicles travel a fixe distance but this can be easily improved by implementing a specific search process, e.g. (Axhausen *et al.*, 1994). Furthermore, the framework can easily accommodate for limited off-street parking capacities and competing strategies between off- and on-street parking as illustrated in section 3.3. It suffices to design a model that determines the search distance with respect to off- and on-street parking occupancies. As already mentioned, it is also possible to define the demands starting from parking by considering a parking duration instead of an exogenous departure rate. Validation studies will be necessary to guaranty that the trip-based simulation outputs with parking are accurate for large and complex networks. The authors are envisioning micro-simulation studies for this but are also looking for real data availability.

Finally, this paper only focuses on car traffic but several extensions to the MFD theory have already been proposed to account for multimodal traffic, e.g. (Zheng and Geroliminis, 2013; Chiabaut, 2015; Zheng and Geroliminis, 2016). The authors are currently working on restating such extensions to fit with the trip-based formulation of the MFD model. They are also working on a multi-reservoir trip-based simulation framework in the spirit of (Aboudolas and Geroliminis, 2013; Ramezani *et al.*, 2015) with parking search to better account for heterogeneous network loadings.

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